注意：

允許學生個人、非營利性的圖書館或公立學校合理使用本基金會網站所提供之各項試題及其解答。**可直接下載而不須申請**。

重版、系統地複製或大量重製這些資料的任何部分，必須獲得財團法人臺北市九章數學教育基金會的授權許可。

申請此項授權請電郵 ccmp@seed.net.tw

Notice:

Individual students, nonprofit libraries, or schools are permitted to make fair use of the papers and its solutions. Republication, systematic copying, or multiple reproduction of any part of this material is permitted only under license from the Chiuchang Mathematics Foundation.

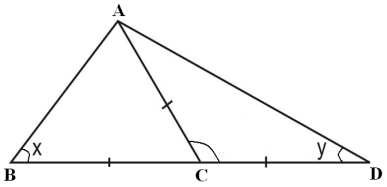
Requests for such permission should be made by e-mailing Mr. Wen-Hsien SUN ccmp@seed.net.tw

SOLUTION OF ESSAY PROBLEMS IMSO 2008

Note:

- For all problems, if the logic is correct but there is a wrong calculation then the deduction due to wrong calculation is at most one.
- Some problems might have some alternative solution. In this case, the marking scheme should refer to the above remark.

1. Find the measure of angle ABC as shown in the following figure, where $AC = CB = CD$, and the measure of angle ADC is 29°.



Solution I:

steps	**Score**
$\angle ACD = 180^o - ((2 \times 29)^o = 122^o$	1
$\angle ACB = 180^o - 122 = 58^o$	1
$\angle ABC = \frac{(180^o - 58^o)}{2} = 61^o$	1

Solution II:

steps	**Score**
$\angle ABC = \angle CAB$ $\angle CDA = \angle CAD$ (isosceles)	1
$\angle BAC + \angle CAD = 90$	1
$\angle ABC = \frac{(180^o - 58^o)}{2} = 61^o$	1

2. In 2008, the price of car A is \$20,000 and car B is \$25,000. Each year, the price of car A decreases by 5% and that of car B decreases by 10%. In what year will car B be cheaper than car A?

Solution:

year	**CAR A** **Decrease** 5% **per year**	**CAR B** **Decrease** 10% **per year**	**Score**
0	20000	25000	0
1	19000	22500	1
2	18050	20250	
3	17147.5	18225	1
4	16290.125	16402.5	
5	**15475.61875**	**14762.25**	

Conclusion: 2013 (score : 1)

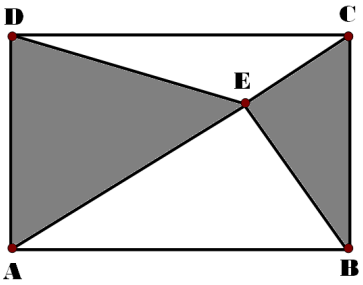
Note :

- Correct Philosophy get 1 point
- Correct calculation get 1 point
- Right conclusion get 1 point

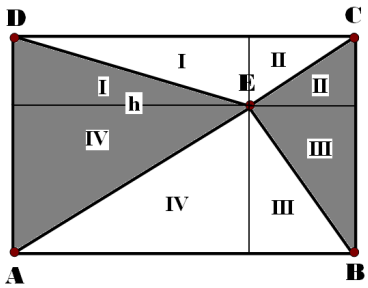
3. The average of 10 consecutive odd numbers is 120. What is the average of the 5 largest numbers? **Solution:**

steps	**Score**
The ten number are: $111, 113, 115, 117, 119, 121, 123, 125, 127, 129$ or The 5 largest numbers are: $121, 123, 125, 127, 129$	1.5
Conclusion: Average of the 5 largest numbers is 125	1.5

4. In the figure $ABCD$ is a rectangle, $AB = CD = 24$ cm and $AD = BC = 5$ cm. What is the area of the shaded region, in cm^2?



Solution:



steps	**Score**
Area triangle $ADE = \frac{5h}{2}$	(1)
Area triangle $BEC = \frac{5(24-h)}{2}$	(1)
Shaded area : $\frac{5h}{2} + \frac{5(24-h)}{2} = 60$	(1)

Note :

- Correct area for each triangle get $(\frac{1}{2})$ point
- Shaded area = unshaded area get 1 point
- Correct conclusion get 1 point

5. You are asked to choose three different numbers from 1 to 10. The sum of the three numbers must be 12. How many choices do you have altogether?

Solution:

The different numbers	**Score**
$1+2+9=12$	
$1+3+8=12$	
$1+4+7=12$	
$1+5+6=12$	$\frac{1}{2}$
$2+3+7=12$	$+\frac{1}{2}$
$2+4+6=12$	$+\frac{1}{2}$
$3+4+5=12$	$+\frac{1}{2}$
Conclusion : We have 7 choice	1

Note :

- 4 solutions get $\frac{1}{2}$ point
- 5 solutions get 1 point
- 6 solutions get $1\frac{1}{2}$ point
- 7 solutions get 2 point
- correct conclusion get 1 point
- incorrect solution $-\frac{1}{2}$ each

6. Five chairs are arranged in a row. A certain five participants must be seated at those chairs. Two of the five participants may not be seated next to each other. In how many ways can we arrange the seating of those five participants?

Solution:
Let the chairs numbers are I, II, III, IV, and V.
Let the name of participants are A, B, C, D, and E.
A and B may not be seated next to each other.

The chair A	**The chair B**	**Ways**	**Score**
I	III, IV, V	$3\times3\times2\times1=18$ ways	$\frac{1}{2}$
II	IV, V	$2\times3\times2\times1=12$ ways	$\frac{1}{2}$
III	I, V	$1\times3\times2\times1=6$ ways	$\frac{1}{2}$
IV	I, II	$2\times3\times2\times1=12$ ways	$\frac{1}{2}$
V	I, II, III	$3\times3\times2\times1=18$ ways	$\frac{1}{2}$

So, we have 72 ways (score : $\frac{1}{2}$) (max 3 points)

Note :

- calculate 5! get 1 point
- calculate $2\times4!$ get 1 point
- correct conclusion get 1 point

7. Find the sum of all numbers from 1 to 500 that are divisible by 5 but not divisible by 2.

Solution:

steps	**Score**
The sequence is: $5, 15, 25, \ldots, 495$ The number of the terms is 50	1 1
The sum is $(500 \times 50)/2 = 12500$	1

Note :

- Sequence divisible by 5 get $\frac{1}{2}$ point
- Sequence divisible by 5 not divisible by 2 get $\frac{1}{2}$ point
- The number of the terms is 50 get 1 point
- Correct answer get 1 point.

8. Let M and N be the areas of a big square and a small square, respectively. The perimeter of the big square is equal to 25 times the perimeter of the small square. What is the ratio of M to N?

Solution:

steps	**Score**
Let x and y be the lengths of one side of a big square and a small square, respectively. Then, $4x = 25 \times 4 \times y$	1
The ratio of the length of the big square to that of the small square : $x : y = 25 : 1$	1
So, $M/N = 625$ or $M : N = 625 : 1$	1

9. Ahmad usually travels from town P to town Q in eight hours. One day, he increased his average speed by 5km per hour so that he arrived 20 minutes earlier. Find his usual average speed, in km per hour.

Solution:

steps	**Score**
If he increased his average speed by 5 km/h so that he arrived 20 minutes earlier or 7 hours and 40 minutes or $\frac{23}{3}$ hours	1
Length PQ = 8× **usual speed** or Length PQ = $\frac{23}{3}\times$ (**usual speed**+5)	1
Conclusion: 8× **usual speed**= $\frac{23}{3}\times$ (**usual speed**+5) $\frac{1}{3}\times$ **usual speed** = $\frac{23}{3} \times 5$ **usual speed** = 115 km/hour	1

10. Nadia wants to make a square using rectangular cards measuring 12.5 cm by 7.5 cm. The cards may not overlap and there may be no gaps between the cards. What is the least number of cards needed?

Solution:

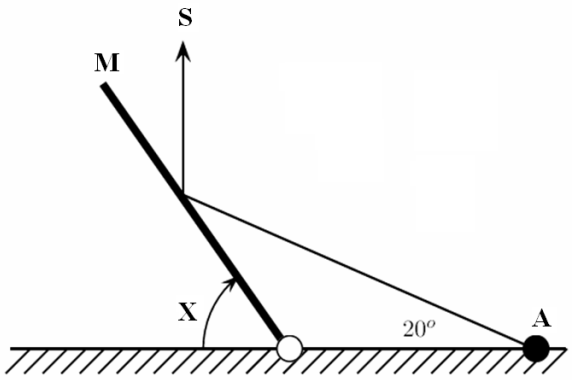
steps	**Score**
The multiple of 7.5 are $7.5; 15; 22.5; 30; 37.5; \ldots$	$\frac{1}{2}$
The multiple of 12.5 are $12.5; 25; 37.5; \ldots$	$\frac{1}{2}$
The LCM of 7.5 and 12.5 is 37.5	$\frac{1}{2}$
The side of the square is is 37.5	$\frac{1}{2}$
The Least number of card $= 3 \times 5 = 15$	1

11. Jenny has a number of books in her bookshelf. She will move all the books to another bookshelf. If she moves 2 books repeatedly then she will have 1 book left. There will also be 1 book left if she moves 3, or 4, or 5 books repeatedly. If she moves 7 books repeatedly, she will have no books left. If the number of all the books is less than 500, find the number of Jenny's books.

Solution:

steps	**Score**
$lcm(2, 3, 4, 5) = 60$	$\frac{1}{2}$
The number of Jenny's books, so that "If she removes the books $2, 3, 4$, or 5 at a time, the remainder is always 1 book left", are: $60 + 1 = 61$, or	1
$120 + 1 = 121$, or $180 + 1 = 181$, or $240 + 1 = 241$, or $300 + 1 = 301$, or $\ldots$	$\frac{1}{2}$
Conclusion: The number of Jenny's books, so that "If she removes the books $2, 3, 4$, or 5 at a time, the remainder is always 1 book left" and "If she removes the books 7 at a time, there are no books left", are: 301	1

12. Light from point A makes an angle of $20°$ to the horizontal plane. It is then reflected by plane mirror M, see the side-view figure on the right. What is the measure of the angle X that makes the reflected light S perpendicular to the horizontal plane?



Solution:

steps	**Score**
Let R be the reflected point at the mirror, O be the rotating point. $\angle ORA = \angle SRM$	1
$\angle ORA = \angle SRM = (180 - 110)/2 = 35$	1
$\angle X = 20 + 35 = 55$	1

13. Consider the sequence $25, 76, 38, 19, 58, 29, \ldots$.
The terms of the sequence are determined by the following rules:

- If a term is even, then the next term is half of it.
- If a term is odd, then the next term is 3 times of it plus 1.

What is the 1000^{th} term?

Solution:

The sequence are

$$\underbrace{25, 76, 38, 19, 58, 29, \ldots, 8}_{21^{th}(1\ point)}, \qquad \underbrace{4, 2, 1, 4, 2, 1, \ldots}_{(+1\ point\ for\ periodic\ subsequence)}$$

So, the 1000^{th} term is 4 score (score 1 point)